Chapter 15 Hypothesis Testing of Population Proportions

I. Introduction

A. The population proportion, first described on page 70, is the average part of a population having a certain characteristic.

1. The **population proportion** (p) follows a binomial probability distribution.
2. It may be expressed as a fraction, decimal, or percentage.
3. Important statistics

Sample Proportion

$$\bar{p} = \frac{\text{\# of successes}}{\text{sample size}} = \frac{x}{n}$$

Interval Estimate for p

$$\bar{p} \pm z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

B. Proportion tests must meet binomial experiment requirements.

1. The experiment must involve two mutually-exclusive outcomes defined as success or failure.
2. Outcomes, which can be counted, must be independent and constant.
3.

n is number of trials	p is probability of success	q, the probability of failure, is 1 - p

C. These proportion tests use the normal approximation of the binomial. This means both np and nq must be ≥ 5 and n must be ≥ 30. The recommended requirement for n varies from 30-100.

II. One-tail testing of one sample proportion

A. Linda is applying for a Flopbuster Video franchise. Flopbuster requires at least 85% of Linda's customers be happy with service at the .05 level of significance. Page 70 sample data indicated 80 of 100 customers were happy with service.

B. Before using the normal approximation to the binomial, the appropriateness of the data must be checked.

1. Both np and nq are ≥ 5 as (100)(.85) = 85 and 100(.15) = 15.
2. The sample size of 100 is ≥ 30.

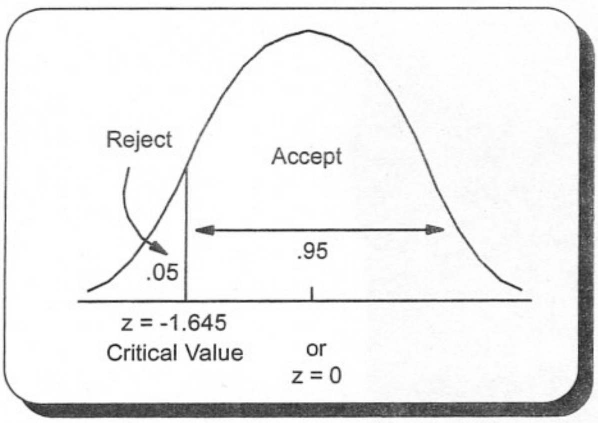
C. The 5-step approach to hypothesis testing

1. The null hypothesis and alternate hypothesis are $H_0: p \geq .85$ and $H_1: p < .85$.
2. The level of significance will be .05 and the critical value of z is -1.645.
3. The relevant statistic will be $\bar{p}$.

$$z = \frac{\bar{p}-p}{\sigma_p} = \frac{\bar{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$$

Note: The standard error of the population proportion is based upon the hypothesized population proportion p (sometimes labeled π), and not the sample proportion.

4. Either of 2 decision rules may be used.
   a. If z from the test statistic is beyond the critical value of z, the null hypothesis will be rejected.
   b. If the p-value is less than the .05 level of significance, the null hypothesis will be rejected.



5. Apply the decision rule.

$$\bar{p} = \frac{x}{n} = \frac{80}{100} = .80$$

$$z = \frac{\bar{p}-p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{.80-.85}{\sqrt{\frac{.85(1-.85)}{100}}} = -1.40$$

Accept H_0 because -1.40 is not beyond -1.645.
Customer satisfaction is ≥ 85%.

The p method yields the same answer.
z = -1.40 → .4192
p = .5000 - .4192 = .0808
Accept H_0 because .0808 >.05.

III. Two-tail testing of one sample proportion

A. When any change is being measured, a two-tail problem exists.

B. If the above problem were stated as a two-tail problem, then $H_0: p = .85$ and $H_1: p \neq .85$ would be appropriate.

C. With a two-tail test, p must be doubled to 2(.0808) = .1616. Accept H_0 because .1616 > .05.